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# INVESTIGATION OF LASER PROPAGATION PHENOMENA

Stuart A. Collins, Jr., et al

Ohio State University

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# INVESTIGATION OF LASER PROPAGATION PHENOMENA

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The Ohio State University

**ElectroScience Laboratory**

Department of Electrical Engineering
Columbus, Ohio 43212

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13. ABSTRACT


This is the first quarterly technical report under a contract entitled "Investigation of Laser Propagation Phenomena". Work during the quarter was concentrated on temporal spectra of phase difference fluctuations of atmospherically degraded spherical waves and on averaging times of optical random data. The temporal spectra derivation considered the case of a horizontal wind transverse to the optical path but with the phase measurement points oriented so that their separ tion vector makes an arbitrary angle with the horizontal. The spectra also included outer scale effects. The averaging time discussion outlined a technique for determining the averaging time of random data while the data is being recorded but in a time much shorter than the total data duration.


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# INVESTIGATION OF LASER PROPAGATION PHENOMENA

S. A. Collins, Jr.
G. W. Reinhardt

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# ABSTRACT

This is the first quarterly technical report under a contract entitled "Investigation of Laser Propagation Phenomena". Work during the quarter was concentrated on temporal spectra of phase difference fluctuations of atmospherically degraded spherical waves and on averaging times of optical random data. The temporal spectra derivation considered the case of a horizontal wind transverse to the optical path but with the phase measurement points oriented so that their separation vector makes an arbitrary angle with the horizontal. The spectra also included outer scale effects. The averaging time discussion outlined a technique for determining the averaging time of random data while the data is being recorded but in a time much shorter than the total data duration.

CONTENTS

## INTRODUCTION

This is the first quarterly technical report under Contract F30602-72-C-0305 titled "Investigation of Laser Propagation Phenomena". This effort is aimed at providing theoretical support to the RADC Laser Propagation Program in the areas of atmospheric propagation phenomena and microturbulence statistics. The efforts are in direct support of the experimental program being conducted at RADC. Two immediate aims are the theoretical description of the temporal spectrum of phase difference fluctuations and theoretical development of methods of determining appropriate averaging times for atmospherically degraded light beams. Work done during the past quarter has concentrated on these two topics. This work is only summarized in this report; the material will be covered in detail in two technical reports to be disseminated at a later date.

The work on temporal spectra is of interest because it provides information on time scales of phase and arrival angle useful to systems designers. More generally it utilizes a simple method of examining experimental data, i.e., the determination of temporal spectra, to give a check on our knowledge of the light-beam turbulence interaction.

In the present report the phase difference temporal spectrum of turbulence degraded light received at two pinholes is considered. Specifically the case when there is a horizontal wind blowing across the beam and the vector between the two pinholes is not necessarily parallel with the ground but makes an arbitrary angle with the horizontal is examined. Outer scale effects are also included.

The averaging time work is important because it establishes the duration of data necessary in order for time averages measured experimentally to agree with the theoretically developed predictions using ensemble averages and a variety of assumptions. In the present report the emphasis is on determining, using a computer-centered technique, the appropriate averaging time while the data is being recorded.

The work performed on these two topics during the past quarter will be summarized respectively in the next two sections.

## TEMPORAL PHASE DIFFERENCE SPECTRA

An analytical expression for the spherical wave phase difference power spectrum has been determined for a horizontal transverse wind and arbitrary angle between the line connecting the phase points and the horizontal using the Von Karmen index spectrum (i.e., including the

finite outer scale effects). This extends previous work (3163-3) where the line between the phase points and wind direction were both horizontal.

In this section the derivation of the analytical expression for the frequency spectrum is summarized. The details of the derivation will be presented in a future report. The final equation of this derivation is evaluated for several limiting cases: small and large normalized separations; and small, intermediate and large normalized frequencies.

The physical picture for the calculation is one of a spherically divergent beam propagating a horizontal distance L to a pair of points in a plane perpendicular to the propagation direction. A wind of constant velocity is blowing across the beam. The difference of the electromagnetic phase of the two points is then measured.

Figure 1 illustrates the coordinate system, where $\phi_1(\rho_1,t)$ and $\phi_2(\rho_2,t)$ are the phases at the two points on a spherical wave front separated by distance $\bar{\rho} = (\bar{\rho}_2-\bar{\rho}_1)$ and a distance L from the spherical wave source. The time-lagged difference in phase between the electromagnetic fields at the phase points is Fourier transformed to yield the desired spectrum.

The spherical wave phase difference spectrum is calculated for any angle between the line connecting the phase points and transverse wind velocity component (v) using the Rytov approximation.

Two cases are of particular interest. The first case is for very small normalized frequencies where the spectrum approaches a constant amplitude except when the wind direction and separation are exactly parallel. The second case occurs when the normalized frequency is large. There the spectrum is orientation independent in contrast to a pronounced orientation dependence for small normalized frequencies.

The phase difference spectrum calculations and experimental measurements are especially useful in the low frequency region where outer scale effects become dominant. In this case phase variance measurements with phase point separation on the order of several outer scale lengths ($L_0$) are replaced with more easily obtained temporal correlation measurements having a time lag comparable to that required for the wind to travel the distance of several outer scale lengths. This technique allows an independent determination of outer scale and directly shows the spectrum of the time variation of the relative phase. This time variation is valuable input for any calculation involving the time variation of quantities dependent on the wave or phase structure functions.

The summary of the derivation starts with an expression for the time lagged phase difference spectrum,

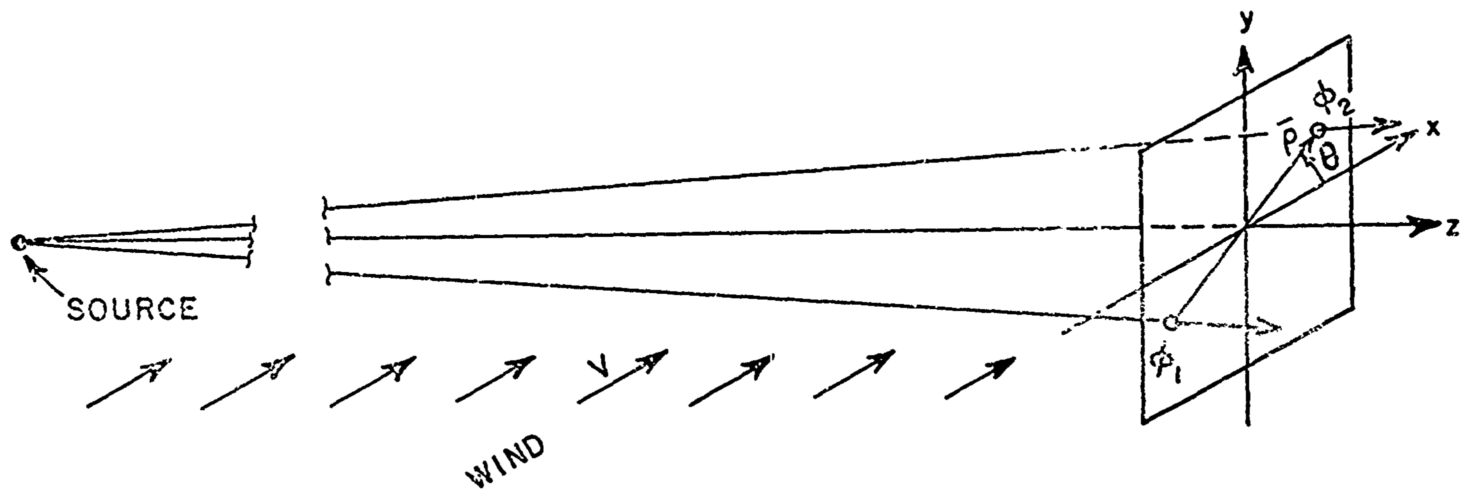


Fig. 1. Phase difference spectrum experiment.

(1) $$W_{\delta s}(\omega) = \mathcal{F}\{\langle(\phi_2(\bar{\rho}_2,\tau+t)-\phi_1(\bar{\rho}_1,\tau+t))(\phi_2(\bar{\rho}_2,t) - \phi_1(\bar{\rho}_1,t))\rangle\}$$

where we use $\mathcal{F}$ to indicate the one dimensional Fourier transform, and brackets indicate the ensemble average. $\bar{\rho}_1$ and $\bar{\rho}_2$ are the phase point coordinates.

Equation (1) is related (Clifford, 1971) to the phase structure function (Carlson, 1967)

(2) $$D_s(\rho) = 8\pi^2k^2 \int_0^\infty \kappa\, d\kappa \left[1-J_0(\kappa\rho)\right] \int_0^L d\eta\, C_n^2(\eta) \left(\frac{L}{\eta}\right)^2 \times \cos^2\left(\frac{L(L-\eta)\kappa^2}{2k\eta}\right) \phi_n^{(0)} \left(\frac{\kappa L}{\eta}\right)$$

and approximately to the wave structure function

(3) $$D_s(\rho) = \begin{Bmatrix} 1. \\ .5 \end{Bmatrix} D_w(\rho) = \begin{Bmatrix} 8 \\ 4 \end{Bmatrix} \pi^2k^2 \int_0^\infty d\kappa\, \kappa(1-J_0(\kappa\rho)) \times \int_0^L d\eta\, C_n^2(\eta)\left(\frac{L}{\eta}\right)^2 \phi_n^{(0)}\left(\frac{\kappa L}{\eta}\right)$$

by employing Taylor's frozen turbulence hypothesis

(4) $$\phi(\rho(\eta), \tau+t) = \phi(\rho(\eta) - v\tau\left(\frac{\eta}{L}\right) \hat{x}, t),$$

where $\eta$ is the longitudinal mean position variable, $(0 < \eta < L)$, and $\hat{x}$ is the unit vector in the x (wind) direction. The index of refraction spectrum is

(5) $$\phi_n^{(0)}(\kappa) = \left(\left(\frac{1.071}{L_0}\right)^2 + \kappa^2\right)^{-11/6}$$

where $\kappa$ is the spatial frequency and $L_0$ is the outer scale.

The resultant integrals consist of several terms that simplify into Delta functions. The remaining integral is written in terms of the normalized variables $\rho$, $\beta$ and R, where

(6a) $\Omega \equiv \frac{2\pi f\rho}{V}$ (6b) $R \equiv \frac{1.071\rho}{L_0}$

(6c) $\beta^2 = \left(1 + \frac{R^2}{\Omega^2}\right)$

It is

$$(7) \quad \frac{W_{\delta s}(\Omega)\left(\frac{V}{\rho}\right)}{8.77\begin{Bmatrix}1.\\.5\end{Bmatrix} k^2 L C_n^2 \rho^{5/3}} = \frac{2\Gamma(11/6)}{\sqrt{\pi}\,\Gamma(4/3)}\ \Omega^{-8/3}\ \beta^{-8/3}$$

$$\times \int_0^\infty d\kappa \left[1 - \frac{\sin(\{\cos\theta + \kappa\beta\sin\theta\}\Omega)}{2\{\cos\theta + \kappa\beta\sin\theta\}\Omega} - \frac{\sin(\{\cos\theta - \kappa\beta\sin\theta\}\Omega)}{2\{\cos\theta - \kappa\beta\sin\theta\}\Omega}\right](1+\kappa^2)^{-11/6}.$$

The integral in Eq. (7) can be evaluated for certain specific cases, depending on frequency, wind velocity, phase point orientation, and outer scale.

Table 1 summarizes the cases for arbitrary orientation angle. The limiting cases are divided into two groups depending on the relative size of $|\Omega\cos\theta \pm (\Omega^2+R^2)^{\frac{1}{2}}\sin\theta|$ and $\sqrt{6}$. A general equation is given for each group. These groups are further simplified into two cases depending on the relative magnitude of the normalized separation and normalized frequency. Table 2 presents the results for large and small separation when the line connecting the phase points is horizontal. Table 3 gives corresponding equations when the wind is perpendicular to the line connecting the phase points.

Figure 2 presents a graph of the limiting cases when the normalized separation, R, is 0.1. This could represent the case for a near earth propagation measurement when the outer scale, $L_0 \approx 2$ meters and the phase point separation is $\approx 20$ cm.

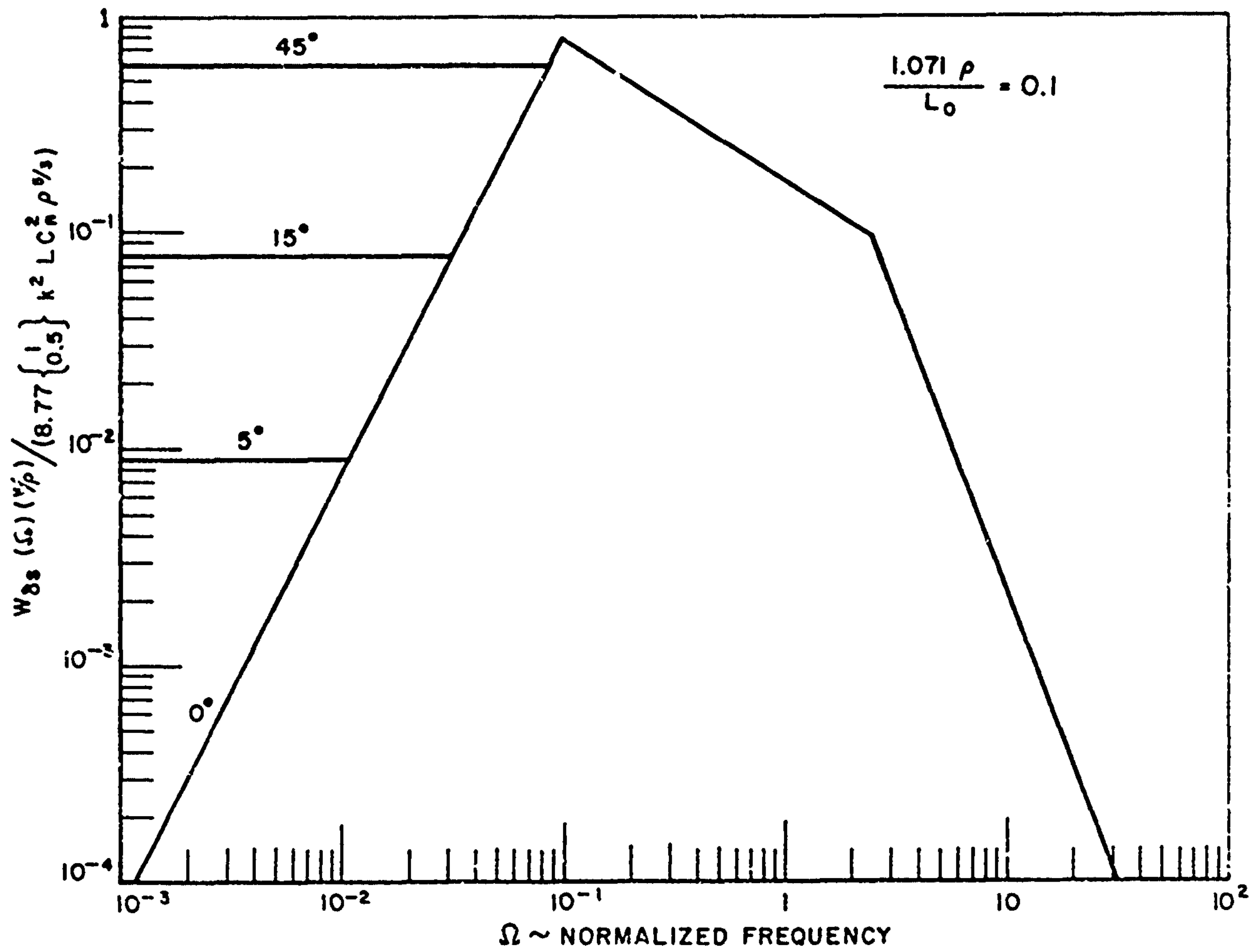


Fig. 2. Limiting form of normalized phase difference spectra versus normalized frequency for several orientation angles.

This figure illustrates the fact that small fluctuations in wind direction could easily mask the $\Omega^2$ dependence of the phase difference frequency spectrum for very small $\Omega$ when the angle $\theta$ is nominally $0^o$.

Referring to Tables 1, 2 and 3, there is a break point in each of the phase difference spectra at $\Omega = R$. This allows a direct determination of the outer scale in terms of break point frequency, phase point separation and wind velocity. Furthermore once $L_0$ is known, one can determine $C_n^2$ from the amplitude of the phase difference spectrum in one or more of the frequency regions where the limiting forms in Tables 1, 2 or 3 apply.

In summary the derivation of an expression for the phase difference spectrum was outlined and a final integral form was given. Tables were presented showing the results of evaluating the integral for limiting cases in each frequency range.

## AVERAGING TIMES

This section will present a survey of the recent work on averaging time determination. A procedure for determination of averaging time while the data is being taken is presented. The developments follow previous work on averaging time in which data was tested after the fact to see if sufficiently long data sections had been obtained.

Here, as in previous work (3163-3) two equivalent averaging time criteria are considered; ergodocity, for which the normalized difference between time and ensemble averages is chosen as the indicator; and stationarity, for which the normalized difference between the data variance and the mean of the variances of many data subsections is chosen as the indicator.

Previously the data was examined after the recording process was completed. The examination indicated whether or not data sufficient for a given precision had been taken. A much more desirable approach is presented here; that of determining while the data is being taken what a sufficient duration will be for a given criterion and indicator.

The method presented assumes the availability of an on-line digital computer fast enough to handle simple arithmetic on the incoming data. An equivalent scheme presented previously (Report 3163-3) used electronic analog techniques. Both schemes rely heavily on developments in the literature: (Lumley and Panofsky, 1964, Bendat and Piersol, 1966, and Charnok and Robinson, 1957).

In the remainder of the section, previous work will be summarized to provide formulae for the real time procedure. Then the basic approach is to determine the autocorrelation integral scale from which the averaging time immediately follows.

TABLE 1

THE ONE SIDED PHASE DIFFERENCE SPECTRUM FOR
ANY ORIENTATION OF PHASE POINTS AND WIND VELOCITY

$$\Omega \equiv \frac{2\pi f\rho}{V} \qquad R \equiv \frac{1.071\rho}{L_0}$$

Small normalized separation $(\theta \neq 0^0)$ $\left|\Omega\cos\theta \pm (\Omega^2+R^2)^{1/2}\sin\theta\right| \ll \sqrt{6}$
and small normalized frequency

$$\frac{W_{\delta s}(\Omega)\left(\frac{V}{\rho}\right)}{8.77\begin{Bmatrix}1.\\ .5\end{Bmatrix}k^2L\,C_n^2\rho^{5/3}} = (\Omega^2+R^2)^{-4/3}\left[\frac{\Omega^2\cos^2\theta}{6} + \frac{(\Omega^2+R^2)\sin^2\theta}{4}\right] \text{General case,}$$

$$= R^{-8/3}\left[\frac{\Omega^2\cos^2\theta}{6} + \frac{R^2\sin^2\theta}{4}\right] \quad R \gg \Omega\,,$$

$$= \Omega^{-2/3}\left[\frac{\cos^2\theta}{6} + \frac{\sin^2\theta}{4}\right] \quad \Omega \gg R\,.$$

Large normalized separation $(\theta \neq 0^0)$ $\left|\Omega\cos\theta \pm (\Omega^2+R^2)^{1/2}\sin\theta\right| \gg \sqrt{6}$
or large normalized frequency

$$\frac{W_{\delta s}(\Omega)\left(\frac{V}{\rho}\right)}{8.77\begin{Bmatrix}1.\\ .5\end{Bmatrix}k^2L\,C_n^2\rho^{5/3}} = (\Omega^2+R^2)^{-4/3} \quad \text{General case,}$$

$$= R^{-8/3} \qquad R \gg \Omega\,,$$

$$= \Omega^{-8/3} \qquad \Omega \gg R\,.$$

TABLE 2

PHASE DIFFERENCE SPECTRUM WHEN THE WIND DIRECTION IS PARALLEL TO THE LINE CONNECTING THE PHASE POINTS

$\Omega = \frac{2\pi f\rho}{v}$ $\qquad$ $R = \frac{1.071\rho}{L_o}$ $\qquad$ $\theta = 0^o$

Small normalized separation: $\underline{\underline{R \ll \sqrt{6}}}$

$$\frac{W_{\delta s}(\Omega)\left(\frac{v}{\rho}\right)}{8.77\begin{Bmatrix}1.\\ .5\end{Bmatrix} k^2 L\ C_n^2 \rho^{5/3}} = \frac{1}{6} R^{-8/3}\Omega^2 \qquad R \gg \Omega$$

$$= \frac{1}{6}\Omega^{-2/3} \qquad \sqrt{6} \gg \Omega \gg R$$

$$= \Omega^{-8/3} \qquad \Omega \gg \sqrt{6}$$

Large normalized separation: $\underline{\underline{R \gg \sqrt{6}}}$

$$\frac{W_{\delta s}(\Omega)\left(\frac{v}{\rho}\right)}{8.77\begin{Bmatrix}1.\\ .5\end{Bmatrix} k^2 L\ C_n^2 \rho^{5/3}} = \frac{1}{6} R^{-8/3}\ \Omega^2 \qquad 6^{3/14} R^{4/7} \gg \Omega$$

$$= \Omega^{-8/3} \qquad \Omega \gg 6^{3/14}\ R^{4/7}$$

TABLE 3

PHASE DIFFERENCE SPECTRUM WHEN THE WIND DIRECTION IS PERPENDICULAR TO THE LINE CONNECTING THE PHASE POINT

$\Omega \equiv \frac{2\pi f\rho}{v}$, $R \equiv \frac{1.071\,\rho}{L_o}$ $\theta = 90^0$

Small normalized separation $\underline{\underline{R << \sqrt{6}}}$

$$\frac{W_{\delta s}(\Omega)\left(\frac{v}{\rho}\right)}{8.77\begin{Bmatrix}1.\\ .5\end{Bmatrix}k^2L\ C_n^2\rho^{5/3}} = \frac{1}{4}R^{-2/3} \qquad R >> \Omega$$

$$= \frac{1}{4}\Omega^{-2/3} \qquad 2 >> \Omega >> R$$

$$= \Omega^{-8/3} \qquad \Omega >> 2$$

Large normalized separation $\underline{\underline{R >> \sqrt{6}}}$

$$\frac{W_{\delta s}(\Omega)\ \frac{v}{\rho}}{8.77\begin{Bmatrix}1.\\ .5\end{Bmatrix}k^2L\ C_n^2\rho^{5/3}} = R^{-8/3} \qquad R >> \Omega$$

$$= \Omega^{-8/3} \qquad \Omega >> R$$

The analytical background will now be outlined. Consider a recording of a random data function, $f(t)$. It is shown in several references: (Lumley and Panofsky, 1964, Bendat and Pierson, 1966, and Davenport and Root, 1958) that $\varepsilon$, the ensemble average mean square difference between the time and ensemble averages normalized to the mean value squares $\langle f\rangle^2$, is related to the averaging time T by

$$(8) \qquad T = \frac{2C(0)}{\varepsilon^2 \langle f\rangle^2} \int_0^T \left(1 - \frac{t}{T}\right)\rho(t)dt$$

where

$C(0)$ = ensemble variance of $f(t)$

$\rho(t)$ = autocorrelation of $f(t)$.

It is generally pointed out that if the autocorrelation becomes substantially zero in a time much less than T, then the second term under the integral is negligible and T is given by

$$(9) \qquad T = \frac{2C(0)}{\varepsilon^2 \langle f\rangle^2} I$$

where I is the integral scale of $\rho$,

$$(10) \quad I = \int_0^\infty \rho(t)dt \quad .$$

Another situation has been considered in the literature (Charnok and Robinson, 1957) in which there are N samples in the complete data section. The data section is divided into m subsections of $s = N/m$ elements.

The variance of each subsection is computed and $\overline{C_S(0)}$ the mean variance of each of the sections is examined. Then by definition

$$(11) \quad \overline{C_s(0)} = \frac{1}{m} \sum_{a=0}^{m-1} \frac{1}{s} \sum_{i=as+1}^{as+s} \left(f_i - \frac{1}{s} \sum_{i=as+1}^{as+s} f_i\right)^2 \quad .$$

By subtracting the complete section mean, $\langle f\rangle$, from both terms in Eq. (11) expanding, rearranging, and writing the resultant expression in terms of the complete section variance

$$C(0) = \frac{1}{m} \sum_{a=0}^{m-1} \frac{1}{s} \sum_{i=as+1}^{as+s} (f_i - \langle f\rangle)^2 \quad ,$$

we get

$$(12) \quad C(0) - \overline{C_s(0)} = \frac{1}{m} \sum_{a=0}^{m-1} \left( \frac{1}{s} \sum_{i=as+1}^{as+s} - \langle f\rangle \right)^2 \quad .$$

Equation (12), as has been indicated previously (3163-3), gives a measure of stationarity. If the section length is sufficiently long and the function is stationary, then each subsection mean will be nearly equal to the complete section mean and the expression in Eq. (12) will be small. The question is how long must each subsection be in order for this to happen. A plot of $C(0) - \overline{C_S(0)}$ versus subsection length s, will indicate the section length for which the difference becomes small, and thus the subsection length for which the function may be regarded as essentially stationary. This requires, however, at least twice as much data as the minimum amount for stationarity (3163-3).

By further rearranging Eq. (12) and defining covariances $C_o(k)$

$$(13) \quad C_o(k) = \frac{1}{m} \sum_{a=0}^{m-1} \frac{1}{(s-k)} \sum_{i=as+1}^{as+s-k} (f_i - \langle f\rangle)\,(f_{i+k} - \langle f\rangle)$$

Equation (12) can be written in the form

$$(14) \quad \frac{C(0) - \overline{C_s(0)}}{C(0)} = \frac{1}{s} \sum_{k=-(s+1)}^{s+1} \left(1 - \frac{|k|}{s}\right) \frac{C_o(k)}{C_N(0)}$$

Taking the limit of the summation as the number of terms becomes large, identifying $C_O(k)/C(0)$ with the autocorrelation $\rho(k)$, and putting $T_s = s\delta t$ and $t_k = k\delta t$ where $\delta t$ is the sampling interval gives

$$(15) \quad \frac{C(0) - \overline{C_s(0)}}{C(0)} = \frac{2}{T_s} \int_0^{T_s} \left(1 - \frac{t_k}{T_s}\right) \rho(t_k)\,\delta t_k$$

The integral in Eq. (15) is identical with that in Eq. (8). If $(t_k)$ becomes small while $t_k/T_S$ is still small, then the integral is identified with the integral scale in Eq. (10) and Eq. (15) becomes

$$(16) \quad \frac{C(0) - \overline{C_s(0)}}{C(0)} = \frac{2I}{T_s}$$

Equation (16) gives the method of finding the integral scale as the data is being taken. The procedure is to plot the function $T_S(C(0)-\overline{C_2(0)})/2C(0)$ versus subsection duration, $T_S$, as data is being received for particular values of $T_N$, the total record duration. For small values of N the function $T_S(C(0) - \overline{C_S(0)})/2C(0)$ is poorly defined. As N becomes sufficiently large the curve asymptotically approaches the value of the integral inner scale.

Once the integral scale is computed, then Eq. (9) can be used to give the averaging time.

The approach of Eq. (16) to the proper integral scale value is limited by two factors. The first is the requirement that the term $t_k/T_S$ in the parentheses under the integral in Eq. (15) be negligible. A data section of only a few integral scales is sufficient for that, and since the complete data section will be the order of a hundred integral scales in duration the requirement is satisfied. The other requirement is that the data sampling rate be sufficiently fast so that the transition from summation to integration in Eq. (15) is sufficiently precise. The digitizing rate can be chosen sufficiently fast to satisfy this requirement, for typical cases when $I \sim 1$ second.

The net result is that the value of the function approaches the value of the integral scale in a time much less than the total data section. The application of Eq. (9) then gives the predicted averaging time.

An experimental check of the above procedure was carried out using optical phase difference data. The data was recorded at the RADC Laser Propagation Range in Verona, N.Y. Figure 3 shows a trace of $T_S(C(0) - \overline{C_S(0)})/2C(0)$ versus data length, $T_S$, for $T_N$ = 10 sec. There it is seen that the ordinate value peaks and then drops to a lower level. The points represent the values $T_N/2$, $T_N/3$, $T_N/4$ secs, etc., corresponds to m = 2, 3, 4, etc. The value at the peak is chosen as a representative conservative value of the integral scale. This value is roughly 0.8 secs which would lead to an averaging time in the range of 80 seconds. Other corroborating studies (3163-3) show that the 80 seconds range is indeed the appropriate one. Further the value was predicted using the first eighth of the data.

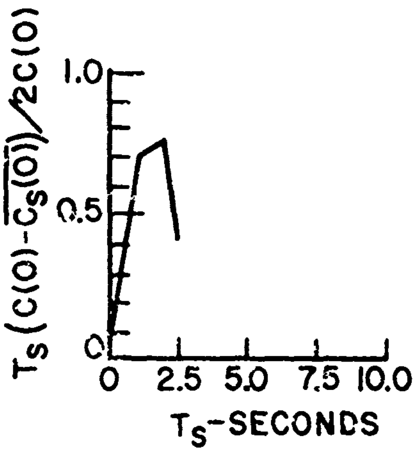


Fig. 3. Real time determination of the integral scale for phase difference data.

In a real time procedure the computer would plot curves equivalent to that of Fig. 3 for successively larger intervals ($T_N$) until a consistent peak is recognized.

To summarize, a real time computer method of predicting averaging time has been outlined and applied to a single set of random optical data. The process predicts an integral scale time from which the averaging time is then obtained. The process will be outlined in more detail and further demonstrated in a report to be released subsequently.

## SUMMARY

During the past quarter, work has concentrated on temporal spectra of phase difference fluctuations of atmospherically degraded spherical waves and on averaging times of optical random data. The temporal spectra considered the case of a horizontal wind transverse to the optical path but with phase points oriented so that their separation vector makes an arbitrary angle with the horizontal. The spectra also included outer scale effects. A final integral was given along with the expressions and graphs for particular limiting cases.

The averaging time discussion outlined a technique for determining the averaging time of random data while the data is being recorded but in a time much shorter than the total data duration. This method assures that a sufficient but economical duration of data is recorded. The procedure assumes the availability of an on-line digital computer.

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